| Benha University<br>Faculty of Engineering – Shoubra<br>Department of Industrial Engineering<br>Course: Mathematics 2 Code: EMP 102 | | Final Exam Group 1<br>Date : May 20 , 2017<br>Answer **All** questions<br>Duration: 2 hours |
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| • The exam consists of one page | | • No. of questions: 4 Total Mark: 40 |

## Question 1

(a)Find **y`** from the following: 6

(i) $y = 2^{x} - 2\sinh x$ (ii) $y = (x^3 + \cosh x)^4$ (iii) $y = \tan^{-1} x + \tanh^{-1} x$

(iv) $y = \log x \,.\, \sin^{-1} x$ (v) $y = \ln x \,.\, \tanh x$ (vi) $y^3 = e^{y} + x.\sinh\ x$

(b)Find the integrals: 6

(i) $\int (x^3 - 4^{x} + 4)\, dx$ (ii) $\int (\frac{1}{x} + \frac{x}{x^2+8})\, dx$ (iii) $\int (\sinh 2x - \cosh x)\, dx$

(iv) $\int \frac{x}{x^2-x-2}\, dx$ (v) $\int \ln x\, dx$ (vi) $\int (3 + \cos^2 2x)\, dx$

## Question 2

(a)Find the area of the region between the curve $y = x^3 - x$ , x-axis, x in [0, 2]. 2

(b)If the region between the curve $y = x^2 + x$ , x-axis, x in [1, 2] is rotated about 4
(i) x-axis (ii)y-axis. Find the volume of the generated solids $V_x$ , $V_y$.

(c)Find the arc length of the curve : $y = \ln x$ , x in [1, 3]. 2

## Question 3

(a)State the definition of the parabola. 2

(b) State the definition of the hyperbola. 2

(c)Separate the lines and find the angle between them : $x^2 - 2xy - 3y^2 = 0$ . 3

(d)Write the equation of circle where the points (3, 2), (1, –1) are ends of diameter. 3

## Question 4

(a)Write the name of the curve : $x^2 - 2xy + y^2 + 4x - 1 = 0$. 2

(b)Find the vertex , focus and sketch the parabola : $y^2 + 8x - 6y - 7 = 0$. 4

(c)Find center, vertices and sketch the hyperbola : $x^2 - 4y^2 - 6x + 16y - 3 = 0$. 4

*Good Luck* *Dr. Mohamed Eid*

# *Model Answer*

**Answer of Question 1**

(a)(i) $y` = 2^x . \ln 2 - 2\cosh x$

(ii) $y` = 4(x^3 + \cosh x)^3 . (3x^2 + \sinh x)$

(iii) $y` = \frac{1}{1+x^2} + \frac{1}{1-x^2}$

(iv) $y` = \frac{1}{\ln 10} . \frac{1}{x} . \sin^{-1} x + \log x . \frac{1}{\sqrt{1-x^2}}$

(v) $y` = \frac{1}{x} . \tanh x + \ln x . \ \mathrm{sech}^2 x$

(vi) $3y^2 y` = e^y . y` + 1 . \sinh x + x . \cosh x$

----------------------------------------------------------------------------------------------------*6- Marks*

(b)(i) $I = \frac{x^4}{4} - \frac{4^x}{\ln 4} + 4x + c$

(ii) $I = \ln x + \frac{1}{2}\ln(x^2 + 8) + c$

(iii) $I = \frac{1}{2}\cosh 2x - \sinh x + c$

(iv) $\int \frac{x}{x^2-x-2} dx = \int (\frac{\frac{2}{3}}{x-2} + \frac{\frac{1}{3}}{x+1}) dx = \frac{2}{3}\ln(x-2) + \frac{1}{3}\ln(x+1) + c$

(v) $\int \ln x \, dx = x \ln x - x + c$

(vi) $I = \int (3 + \frac{1}{2}(1 + \cos 4x) dx = \frac{7}{2}x + \frac{1}{8}\sin 4x + c$

----------------------------------------------------------------------------------------------------*6- Marks*

**Answer of Question 2**

(a)From $x^3 - x = 0$ , we get x = 0, 1 , – 1. We see that 1 inside the interval and 0, – 1 outside the interval [0, 2].

Then $A = \int_0^1 ( x^3 - x) dx + \int_1^2 ( x^3 - x) dx = | -\frac{1}{4} | + \frac{9}{4} = 2.5$

----------------------------------------------------------------------------------------------------*2- Marks*

(b)(i) $V_x = \pi \int_1^2 (x^2 + x)^2 \, dx = 16.03 \, \pi$

(ii) $V_y = 2\pi \int_1^2 x(x^2 + x) \quad dx = 12.16 \, \pi$

----------------------------------------------------------------------------------------------------*4- Marks*

(c)The arc length $A = \int_1^3 \sqrt{1 + \frac{1}{x^2}} dx = 2.3$

----------------------------------------------------------------------------------------------------*2- Marks*

**Answer of Question 3**

(a)Parabola------------------------------------------------------------------------------------------2- *Marks*
(b)Hyperbola-----------------------------------------------------------------------------------------2- *Marks*

(c) $x^2 - 2xy - 3y^2 = (x+y)(x-3y) = 0$

The two lines are : x + y = 0 , x – 3y = 0 and $\tan\theta = \frac{-1-\frac{1}{3}}{1-\frac{1}{3}} = 2$

------------------------------------------------------------------------------------------------------3- *Marks*

(d)The circle is : $(x-3)(x-1) + (y-2)(y+1) = 0$ Or $x^2 + y^2 - 4x - y + 1 = 0$

------------------------------------------------------------------------------------------------------3- *Marks*

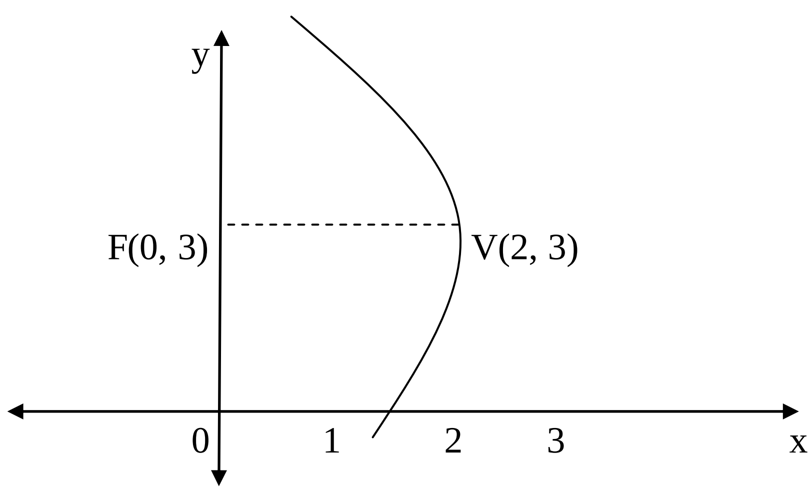
**Answer of Question 4**

(a) $\Delta = \begin{vmatrix} 1 & -1 & 2 \\ -1 & 1 & 0 \\ 2 & 0 & -1 \end{vmatrix} = -4 \quad and \quad a.b = 1 = h^2$. Then it is parabola.

------------------------------------------------------------------------------------------------------2- *Marks*

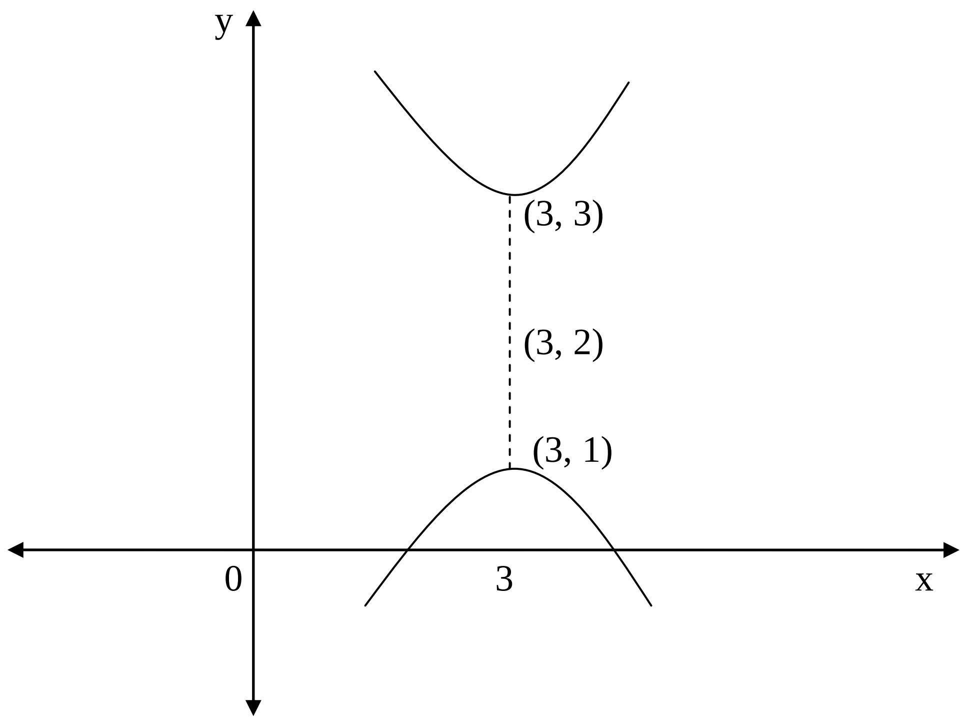
(d)From : $y^2 + 8x - 6y - 7 = 0$ , we get $(y-3)^2 = -8(x-2)$

The vertex is (2, 3) , a = – 2, horizontal.




------------------------------------------------------------------------------------------------------4- *Marks* (e)From : $x^2 - 4y^2 - 6x + 16y - 3 = 0$ , we get $\frac{(y-2)^2}{1} - \frac{(x-3)^2}{4} = 1$

The center is (3, 2) , b = 1, vertical.




------------------------------------------------------------------------------------------4- *Marks*

*Dr. Mohamed Eid*